Relations and the Coordinate plane

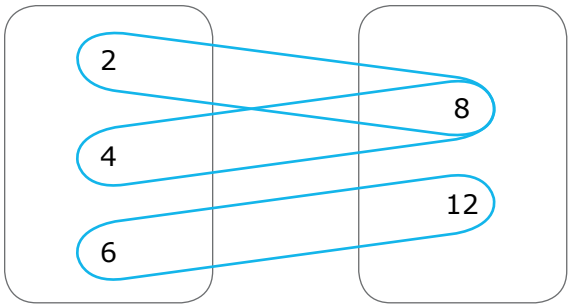
After completing this lesson you will be able to:

- Create a mapping diagram of a relation
- Identify parts of the Coordinate Plane
- Graph and generate ordered pairs of a relation
- Generate a table of values
- Determine if a relation is a function

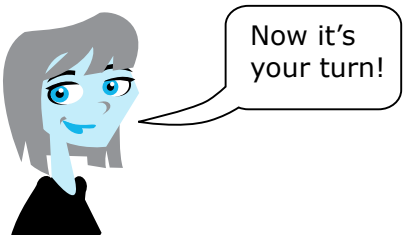
Pairing of input and output values for a situation is called a **mapping**. The set of input values is called the **domain** and the set of output values is the called the **range**.

Relations between two quantities can be represented in many ways, including mapping diagrams, tables, graphs, equations, and verbal descriptions.

An example of a **mapping diagram** is shown here:



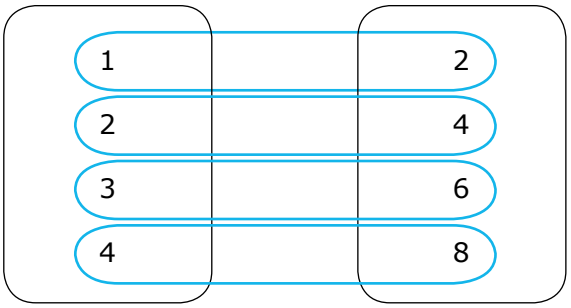
This is called a relation and is a pairing of two quantities. The input value 2 is mapped to the output of 8 and this may be written as the ordered pair (2,8). The other ordered pairs in this mapping are (4, 8) and (6, 12). The relation is {(2, 8), (4, 8), (6, 12)}.



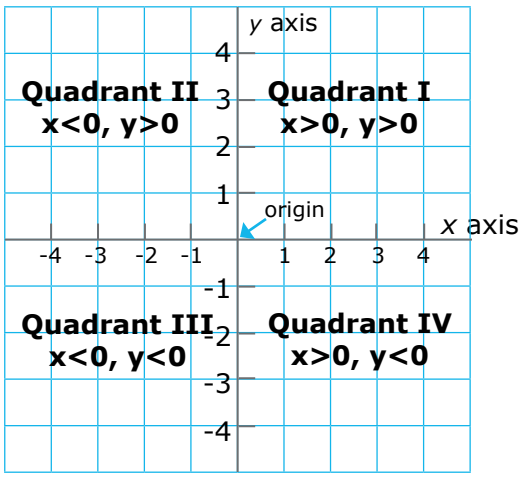
Problem

Create a mapping diagram for the following relation:
((1, 2), (2, 4), (3, 6), (4, 8))

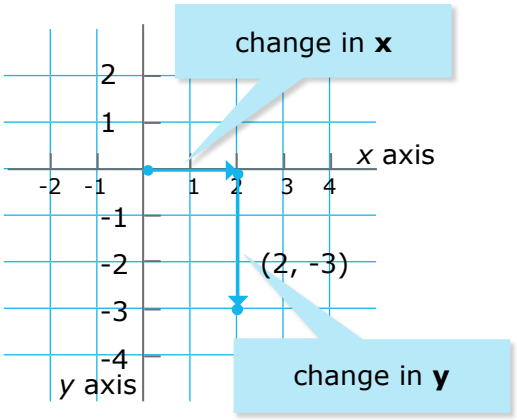
Solution:



A relation may also be graphed as a set of ordered pairs (x, y) on the **coordinate plane**. The coordinate plane is divided into four **quadrants** by the **x-axis** and the **y-axis**. The axes intersect at the origin. The ordered pairs of a relation may be graphed by finding the x value and its corresponding y-value on the coordinate plane.

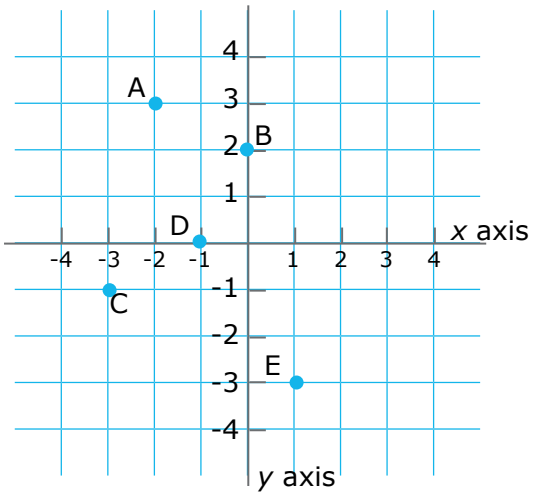


The **origin** is the point (0, 0) and to graph any other point in a relation you begin at that point. To graph the point (2, -3) we would begin at the origin, move 2 units to the right along the x-axis and 3 units down in the direction of the y-axis.



Problem

Identify the points on the coordinate plane below:



Solution:

A (-2, 3) B (0, 2) C (-3, -1) D (-1, 0) E (1, -3)

To learn more about this in depth, go to the Concept Capsule titled **Graphing in Two Dimensions**.

In your Workbook, try **Questions 1-15** for this lesson.

Generating a Table of Values

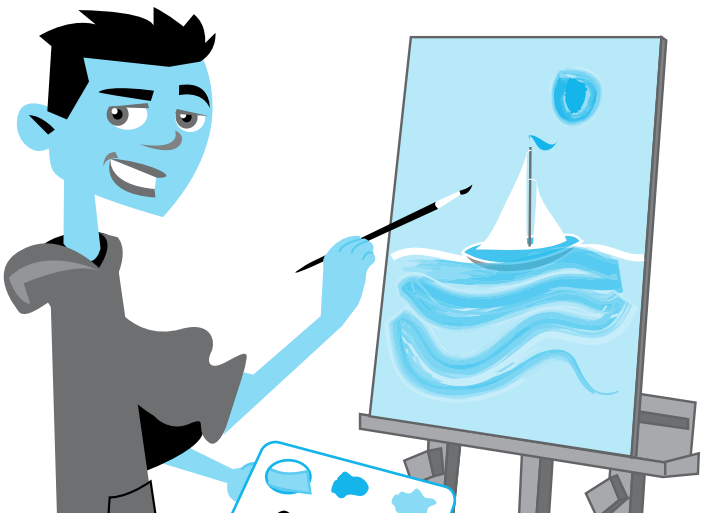
The income Cameron would make if he gets a job that pays $6.50 per hour is a type of relation.

Let's start by generating a table of values for the problem. If Cameron works 1 hour, he will be paid $6.50. If he works 2 hours then he will be paid $6.50 · 2 hours or $13 and so on.

Hours worked	1	2	3	4	5	6	7
Pay	6.50	13	19.50	26	32.50	39	45.50

We can use patterns to predict the outcomes for various numbers of hours worked.

Example 1



Daksha can paint at a rate of 10 square metres per hour. Create a table of values for the number of square meters painted in each hour given.

1. The input values (x – values) will be the number of hours worked.
2. The output values (y)will be the number of square meters painted.
3. To find the y for each x you will multiply by 10 square meters.

Hours worked	1	2	3	4	5	6	7
Pay	10	20	30	40	50	60	70

Example 2

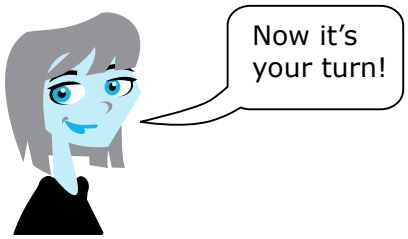
Generate the y values for the equation: $y = 2x - 3$

given the x values in the table.

$y = 2(-2) - 3 = -7$ Substitute x with -2 and simplify

Repeat process for each input or x-value.

x	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3



Problem

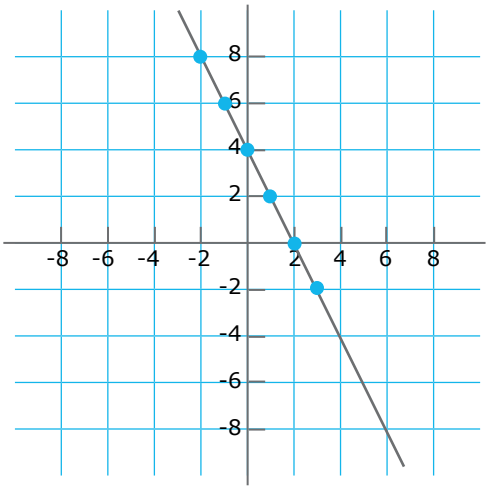
Generate the table of values for the relation defined by $y = -2x + 4$ and graph the relation.

Solution

Step 1: One possible table of values:

x	-2	-1	0	1	2	3
y	8	6	4	2	0	-2

Step 2: Plot the values in the table and graph the line:



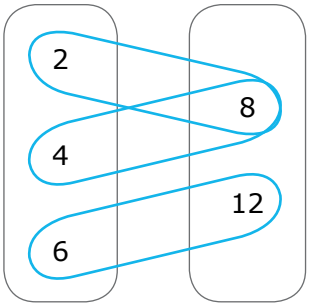
In your Workbook, try **Questions 16-22** for this lesson.

Determining if a Relation is a Function

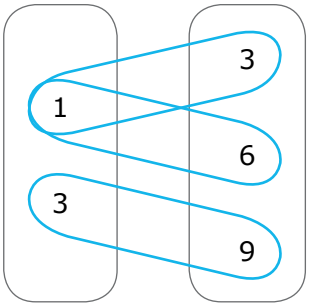
Some relations are called functions. A **function** is a relation that has exactly one *output* for every *input*. Using the definition we can identify functions from mapping diagrams and ordered pair listings of relations.

Example 3

a. Is each of the following relations a function?



Yes. Each Input has one inputs and there are no inputs that have two lines going to the output.



No. One of the input values has more than one output value.

So basically, if an input has more than one oval coming from it then this will not be a function.

b. Is each of the following a function?
Hint: remember that the x-values are the inputs.

((1, 3), (-1, 3), (2, 3), (4, 3))	Yes. Even though each input has the same output, no one input has more than one output.
((3, -2), (4, -1), (4, -3))	No. The input 4 has outputs of -1 and -3.

Another method that may be used to identify functions is visual. By graphing a relation on the coordinate plane you can use a test called the Vertical Line Test to determine if it is a function.

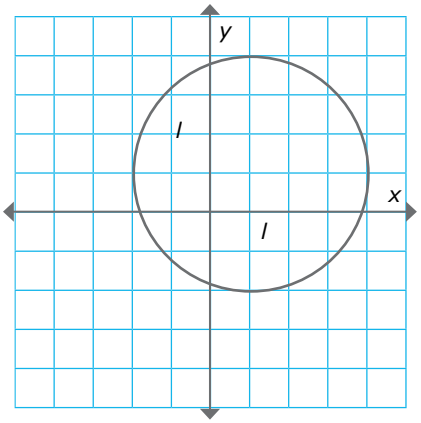
The Vertical Line Test

If a vertical line intersects the graph of a relation in more than one point then it is not a function.

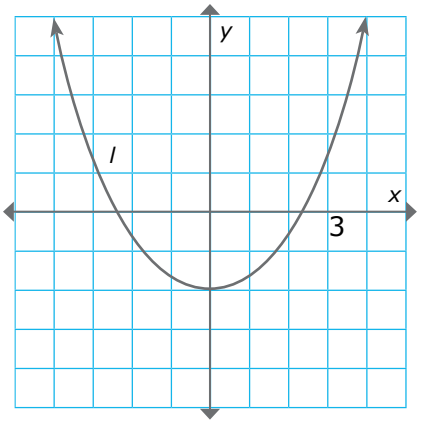
Example 3

Use the vertical line test to determine if the relation is a function:

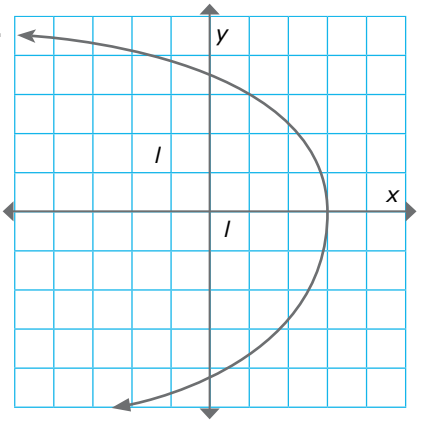
a.



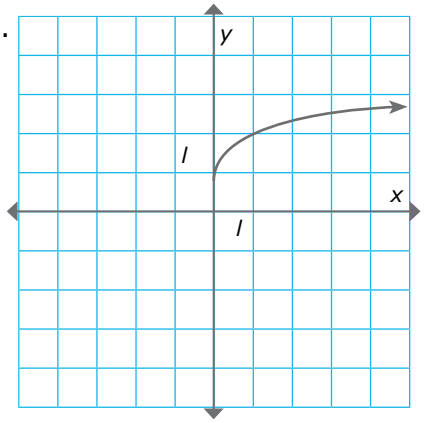
b.



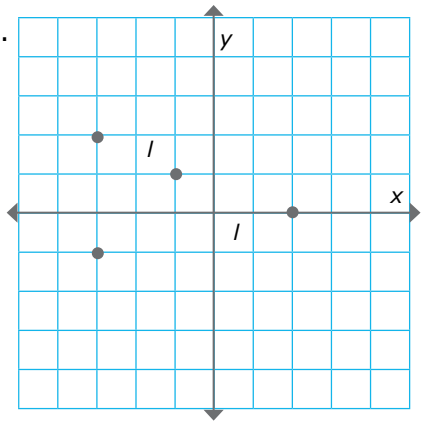
c.



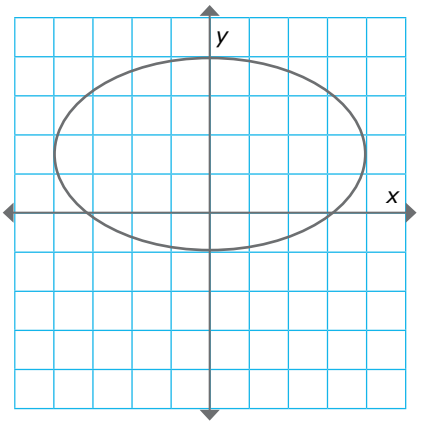
d.



e.



f.



Solution

The graphs a, c, and f are not functions. If you put a vertical line over the graph then you would cross them in more than one place, therefore they fail the vertical line test. It is not possible for a vertical line to pass through b, d, and e more than once; therefore they do pass and are functions.

In your Workbook, try **Questions 23-34** for this lesson.